Simple and Efficient Single Round Almost Perfectly Secure Message Transmission Tolerating Generalized Adversary

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Encryption Schemes

	Must share a secret-key	Don’t share a secret-key
Computational	SKE	PKE
Unconditional	One-time pad	

Does there exist ?

	Must share a secret-key	Don’t share a secret-key
Computational	SKE	PKE
Unconditional	One-time pad	???

Yes

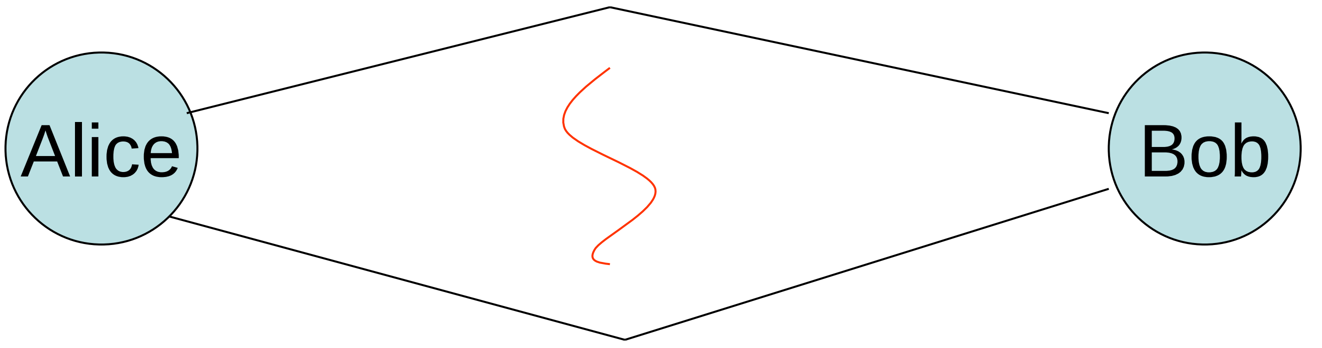
- (1975) Wyner

  Wire-tap channel model
- (1984) Bennett and Brassard

  BB84
- (1993) Dolev, Dwork, Waarts and Yung

  Network model

In the model of DDWY



- Alice and Bob are a part of a network
- There are n channels between them
- Adversary can corrupt (listen and forge)

    at most t channels

Indeed, in Internet

- There are many channels
  between A and B
- No adversary can corrupt all the routers

A scheme should satisfy

- (Perfect Privacy)

  Adversary learns no information on the secret message s
- (Perfect Reliability)

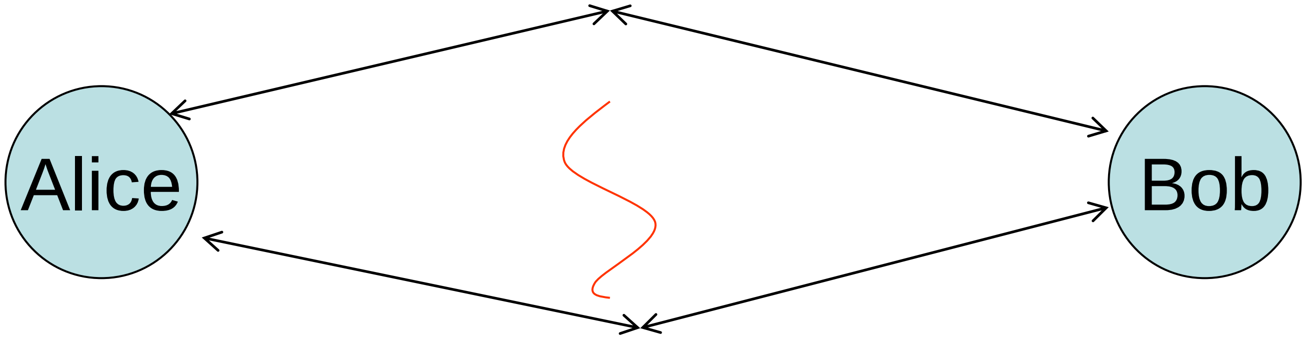
  Bob can receive s correctly

  (Adversary cannot forge s)

PSMT denotes

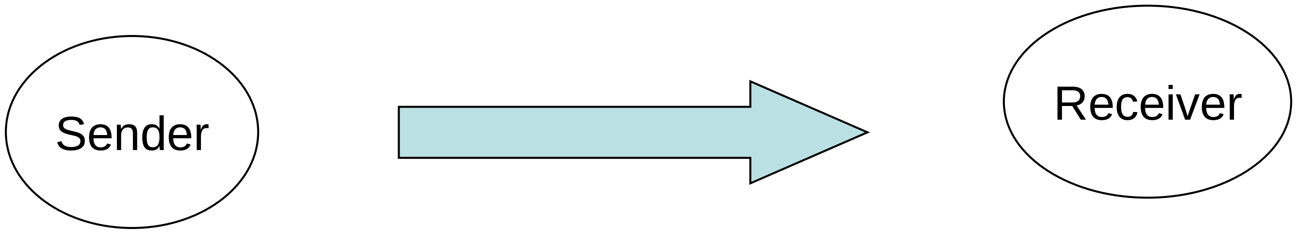
- Perfectly
- Secure
- Message
- Transmission
- Scheme

We consider an Undirected Network

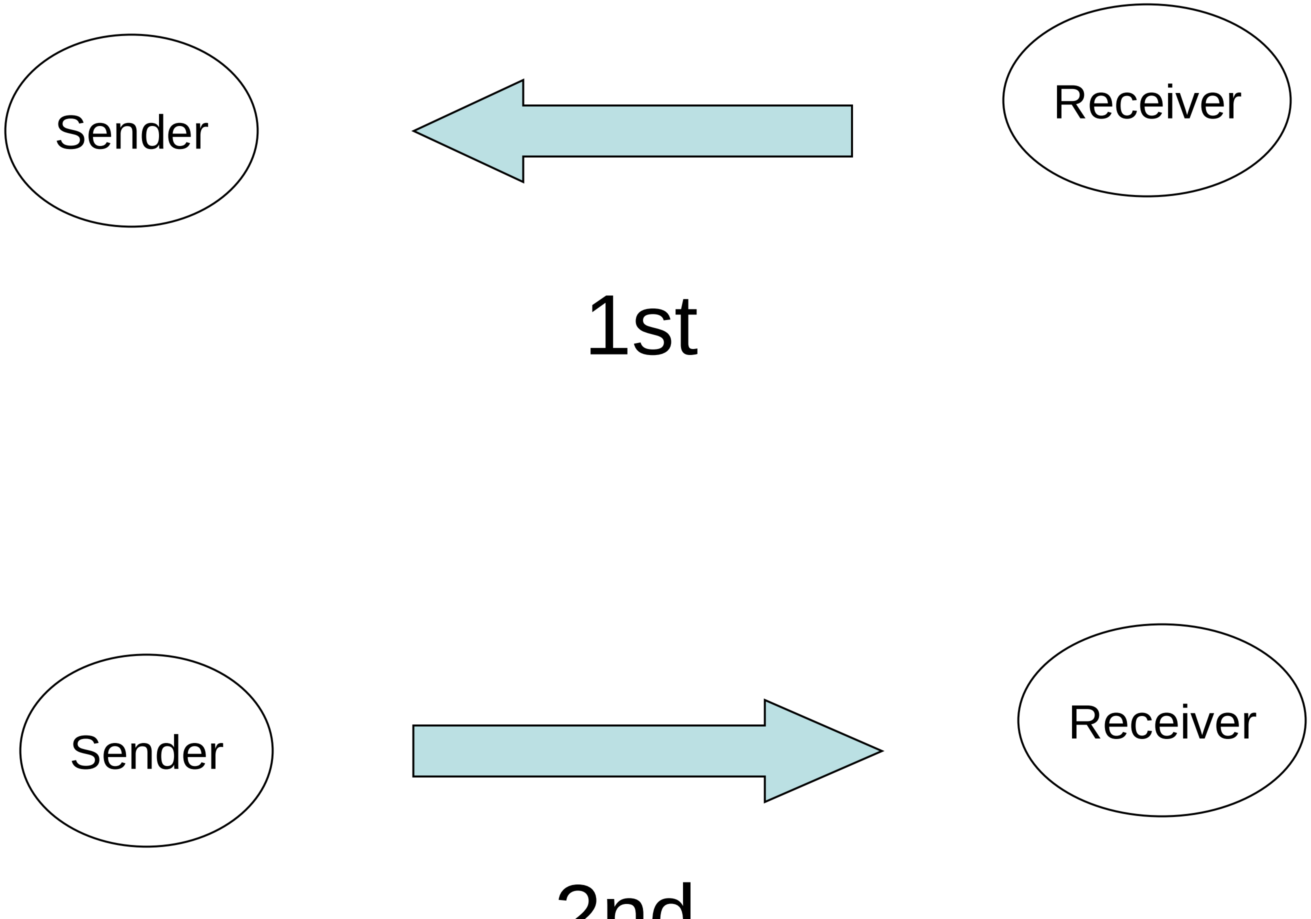


- Each channel is two-way

1 Round Protocol



2 Round Protocol



PSMT exists

1-round	iff $n \geqq 3t+1$
2-round	iff $n \geqq 2t+1$

where the adversary can corrupt
t out of n channels.

Almost PSMT

requires

- (Perfect Privacy)

  Adversary learns no information on the secret message s

- (Almost Perfect Reliability)

  Pr[Bob can receive s] > 1- ε

If $n \geqq 2t+1$,

PSMT requires	2 rounds
Almost PSMT requires	only 1 round

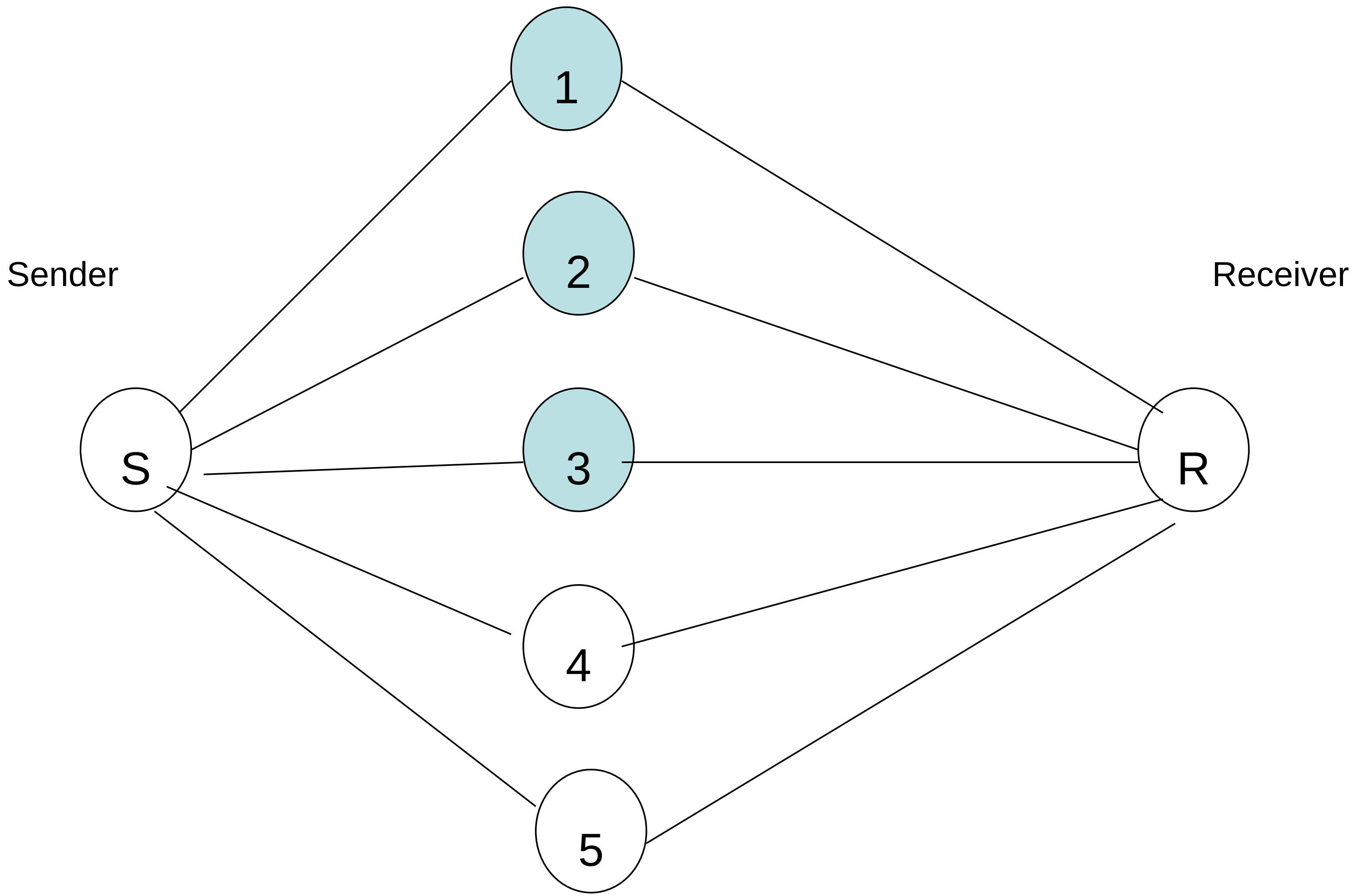
So far

	PSMT	**Almost PSMT**
Threshold adversary	We have seen	We have seen
How about General adversary	?	?

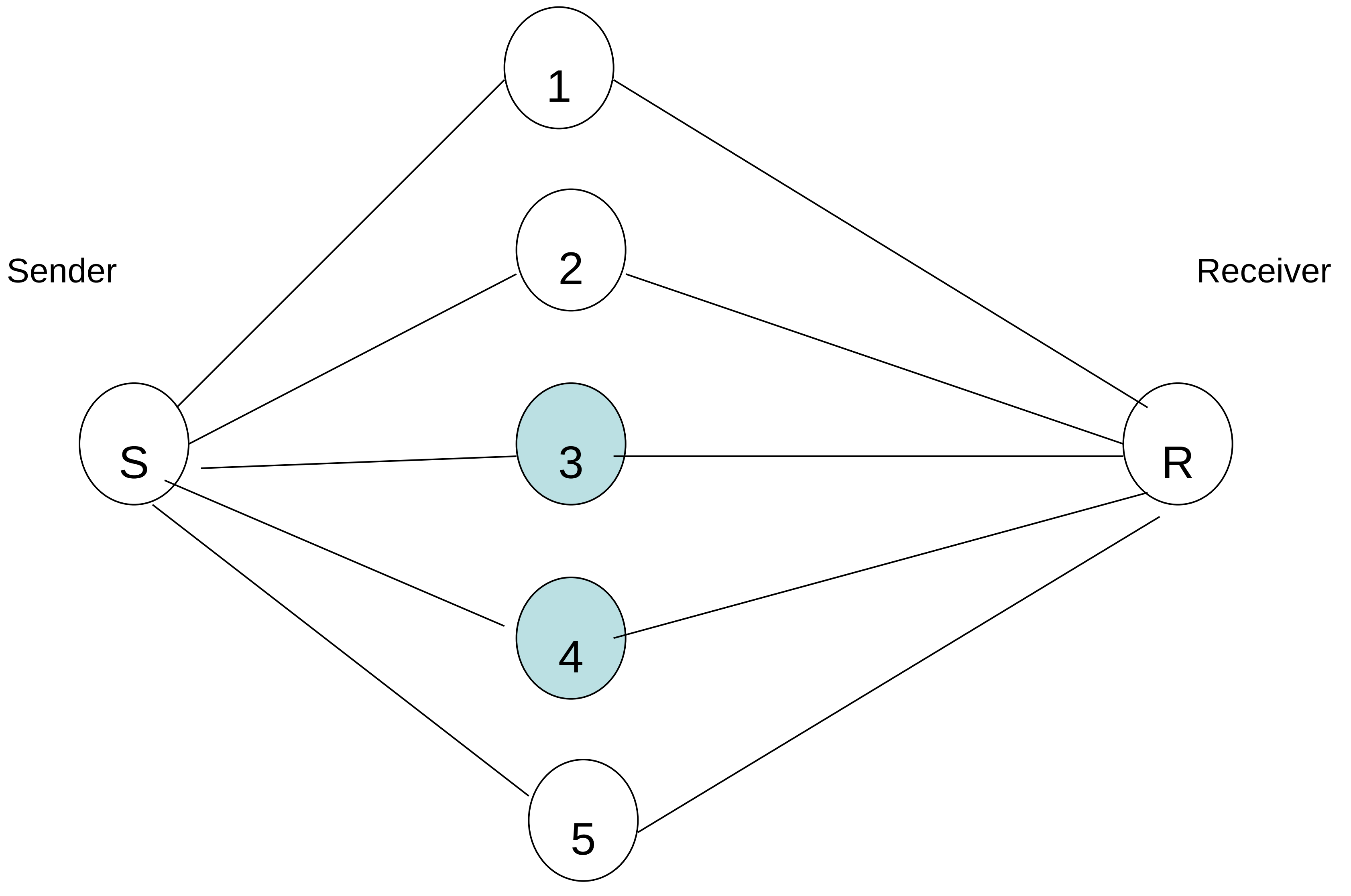
Desmedt et at.

- Threshold adversaries are not realistic
- when dealing with computer viruses,
- such as
- the I LOVE YOU virus
- and the Internet virus/worm
- that only spread to
- Windows, respectively Unix.

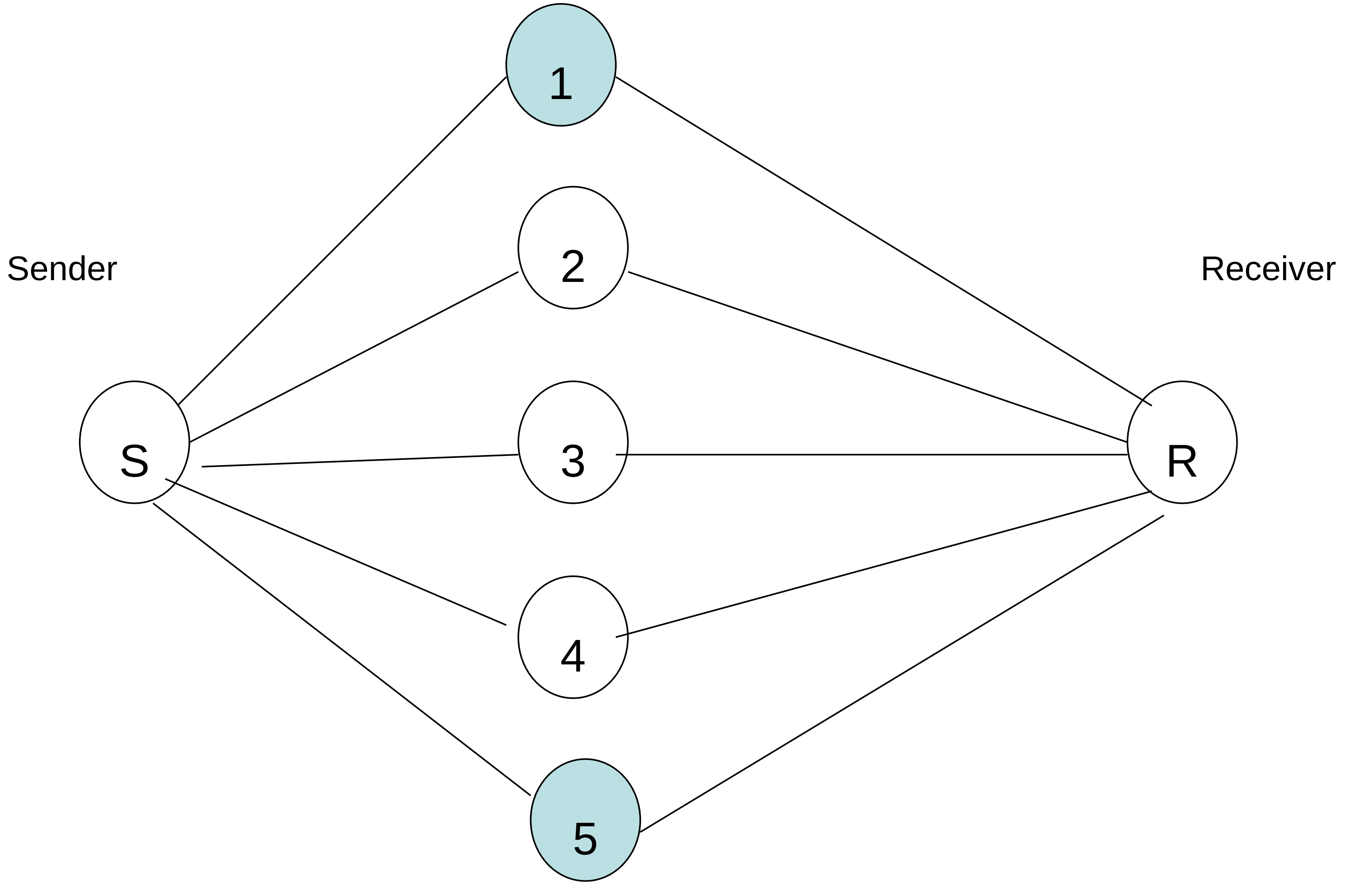
{1,2,3} use Windows



{3,4} use UNIX



{1,5} use TRON



Adversary can corrupt

$B_1=\{1,2,3\}$ or $B_2=\{3,4\}$ or $B_3=\{1,5\}$.

- Let

  $\Gamma=\{B_1, B_2, B_3\}$

- Such Γ is called an adversary structure.

Monotone

- We say that Γ is monotone
  if $B \in \Gamma$ and $B' \subset B$, then $B' \in \Gamma$
- For example.
  if an adversary can corrupt B={1,2,3},
  then she can corrupt B'={1,2} clearly.
- In what follows,
  we assume that Γ is monotone

Hirt and Maurer

- Introduced adversary structure
  in the context of multiparty protocols
- They generalized

  $n \geqq 2t+1$ to Q^2 adversary structure

  $n \geqq 3t+1$ to Q^3 adversary structure

Γ satisfies Q^2

- If

  $B_i \cup B_j \neq \{1, \cdots, n\}$

- for any $B_i, B_j \in \Gamma$

$\Gamma=\{B_1, B_2, B_3\}$

- Such that

  $B_1=\{1,2,3\}$, $B_2=\{3,4\}$, $B_3=\{1,5\}$.

- is Q^2 because

  $B_1 \cup B_2 = \{1,2,3,4\} \neq \{1, \cdots, 5\}$

  $B_1 \cup B_3 = \{1,2,3,5\} \neq \{1, \cdots, 5\}$

  $B_2 \cup B_3 = \{1,3,4,5\} \neq \{1, \cdots, 5\}$

Γ satisfies Q^3

- If

  $$B_i \cup B_j \cup B_k \neq \{1, \cdots, n\}$$

- for any $B_i, B_j, B_k \in \Gamma$

For general adversaries,

1-round PSMT	iff Γ satisfies Q^3
2-round PSMT	iff Γ satisfies Q^2

	PSMT	Almost PSMT
Threshold adversary	We have seen	We have seen
General adversary	We have seen	

? is

	PSMT	Almost PSMT
Threshold adversary	We have seen	We have seen
General adversary	We have seen	?

For the ?

- Patra, Choudhary, Srinathan, and Rangan
- showed an almost PSMT for Q^2.

However,

- At least 3 rounds
- Exponential time

This paper shows

- An efficient 1 round almost PSMT for Q^2

	# of rounds	Efficiency
Patra et al.	At least 3	Inefficient
Our scheme	1	Efficient

Hence
for Q^2 adversary structure,

PSMT requires	2 rounds
Almost PSMT requires	only 1 round (This paper)

In a Secret Sharing Scheme

- For a secret s,

  Dealer computes a share vector

  $$(\text{share}_1, \cdots , \text{share}_n),$$

  and gives share_i to player P_i

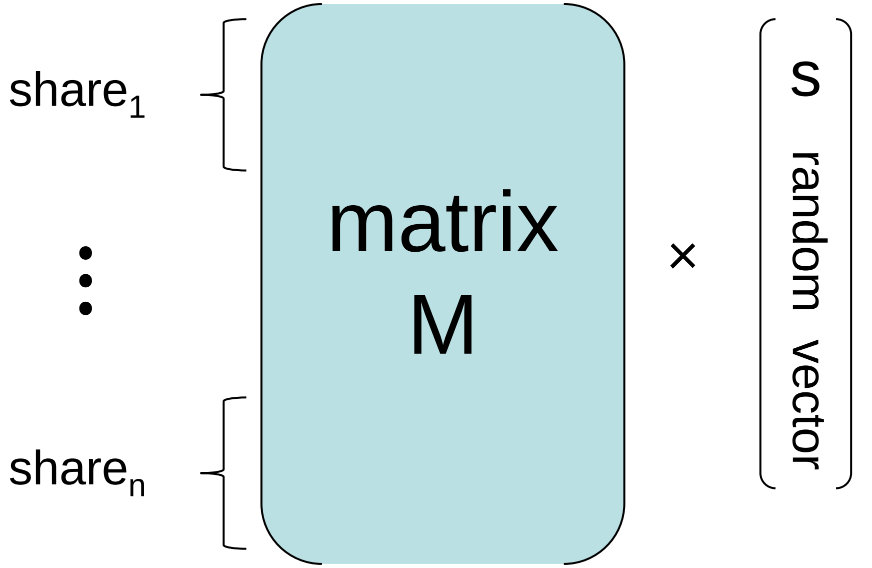
Proposition

For any adversary structure Γ,
there exists a linear secret sharing scheme (LSSS)
such that

- if $B \in \Gamma$, then B has no information on s
- if $A \notin \Gamma$, then A can reconstruct s

We call it an LSSS for Γ

In a LSSS



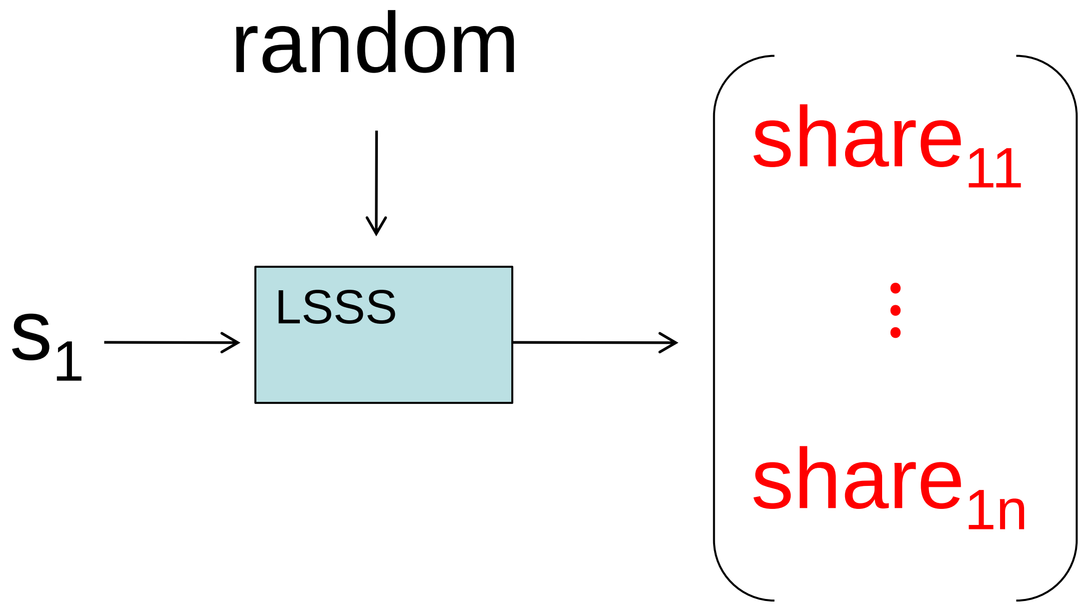
A share vector is computed by multiplying
(s, random vector)
to some matrix M

In our 1 round almost PSMT

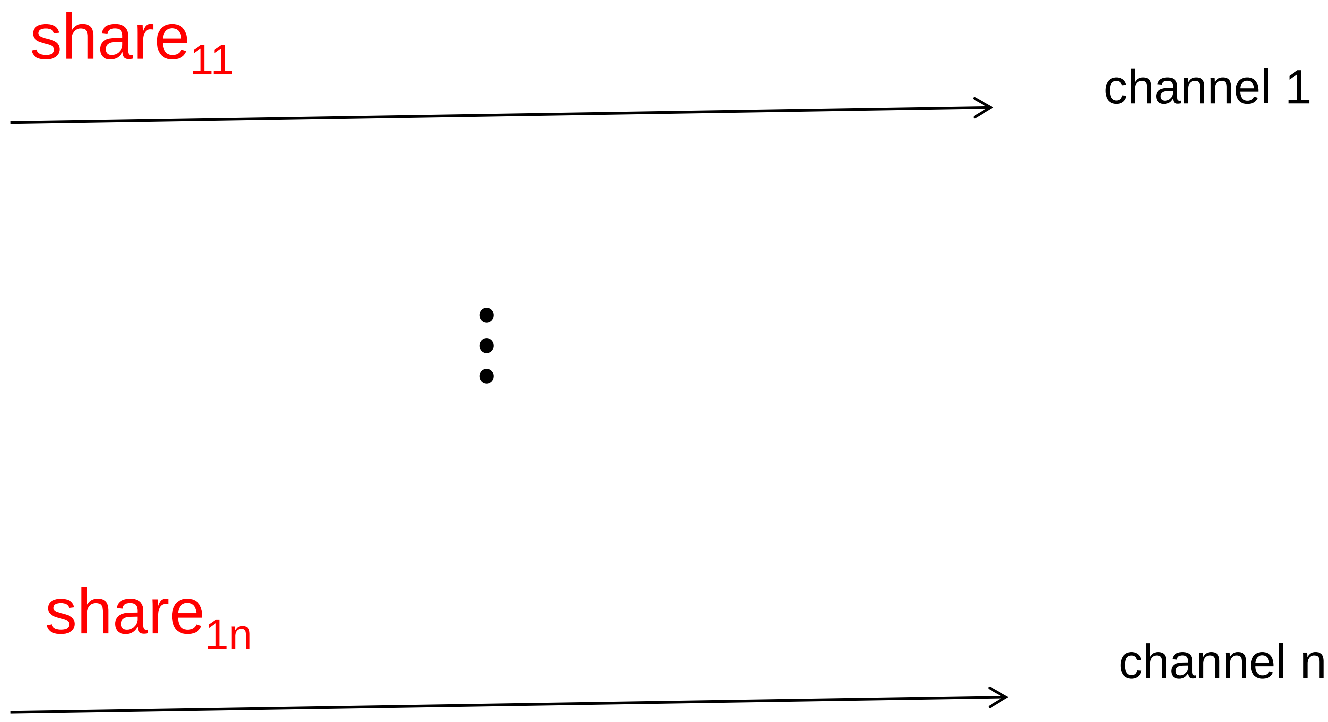
- We are given:
  - An adversary structure Γ satisfying Q^2 condition

- We then use an LSSS for this Γ
- Suppose that the sender wants to send a message $(s_1, \cdots, s_L)$ to the receiver.

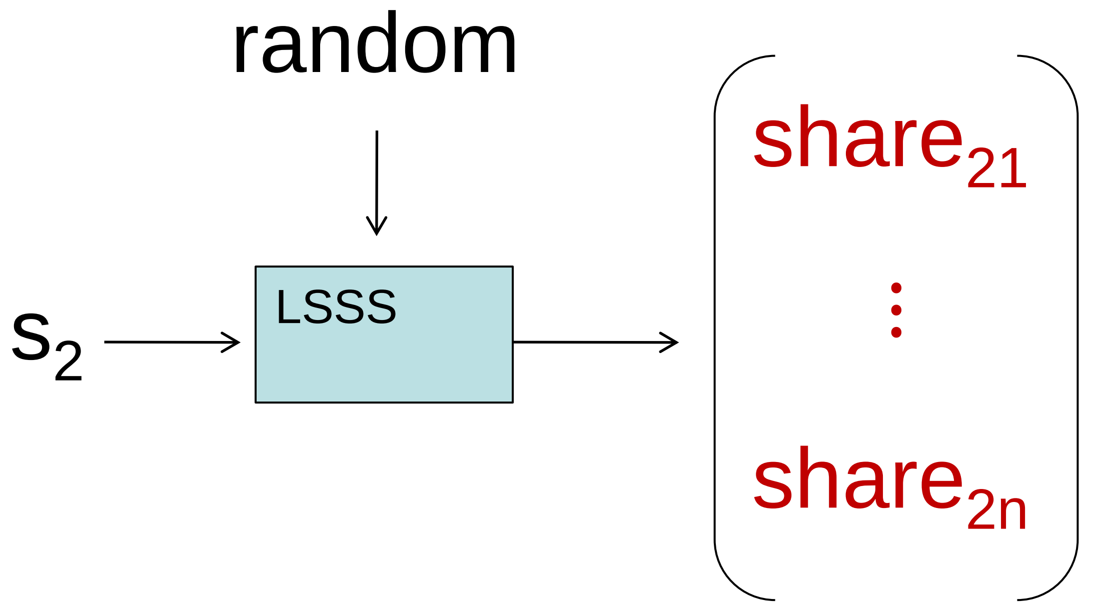
For s_1, sender computes



Sender sends to the receiver



For s_2, sender computes



Sender sends to the receiver

$share_{11}$, $share_{21}$ channel 1

⋮

$share_{1n}$, $share_{2n}$ channel n

and so on

$share_{11}$, $share_{21}$, ⋯, $share_{L1}$ → channel 1

⋮

$share_{1n}$, $share_{2n}$, ⋯, $share_{Ln}$ → channel n

Adversary learns no information on each s_i

- because Adv can listen to only a subset of channels $B \in \Gamma$
- From our property of the LSSS, $B \in \Gamma$ give no information on s_i

However

- Adv may forge the shares in B $\in \Gamma$
- To detect this forgery,

  Sender sends some additional authentication information.

To authenticate

$share_{11}, share_{21}, \cdots, share_{L1}$ → channel 1

⋮

$share_{1n}, share_{2n}, \cdots, share_{Ln}$ → channel n

We consider polynomials

$p_1(x) = share_{11} + share_{21}\, x + \cdots + share_{L1}\, x^{L-1}$

→ channel 1

⋮

$p_n(x) = share_{1n} + share_{2n}\, x + \cdots + share_{Ln}\, x^{L-1}$

→ channel n

To authenticate $p_1(x)$

$p_1(x) = share_{11} + share_{21}\, x + \cdots + share_{L1}\, x^{L-1}$

channel 1

random α_2 and $p_1(\alpha_2)$

channel 2

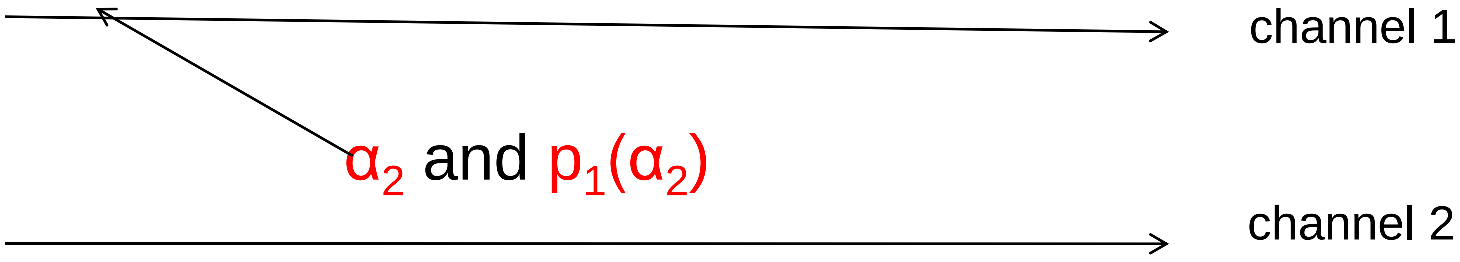
⋮

random α_n and $p_1(\alpha_n)$

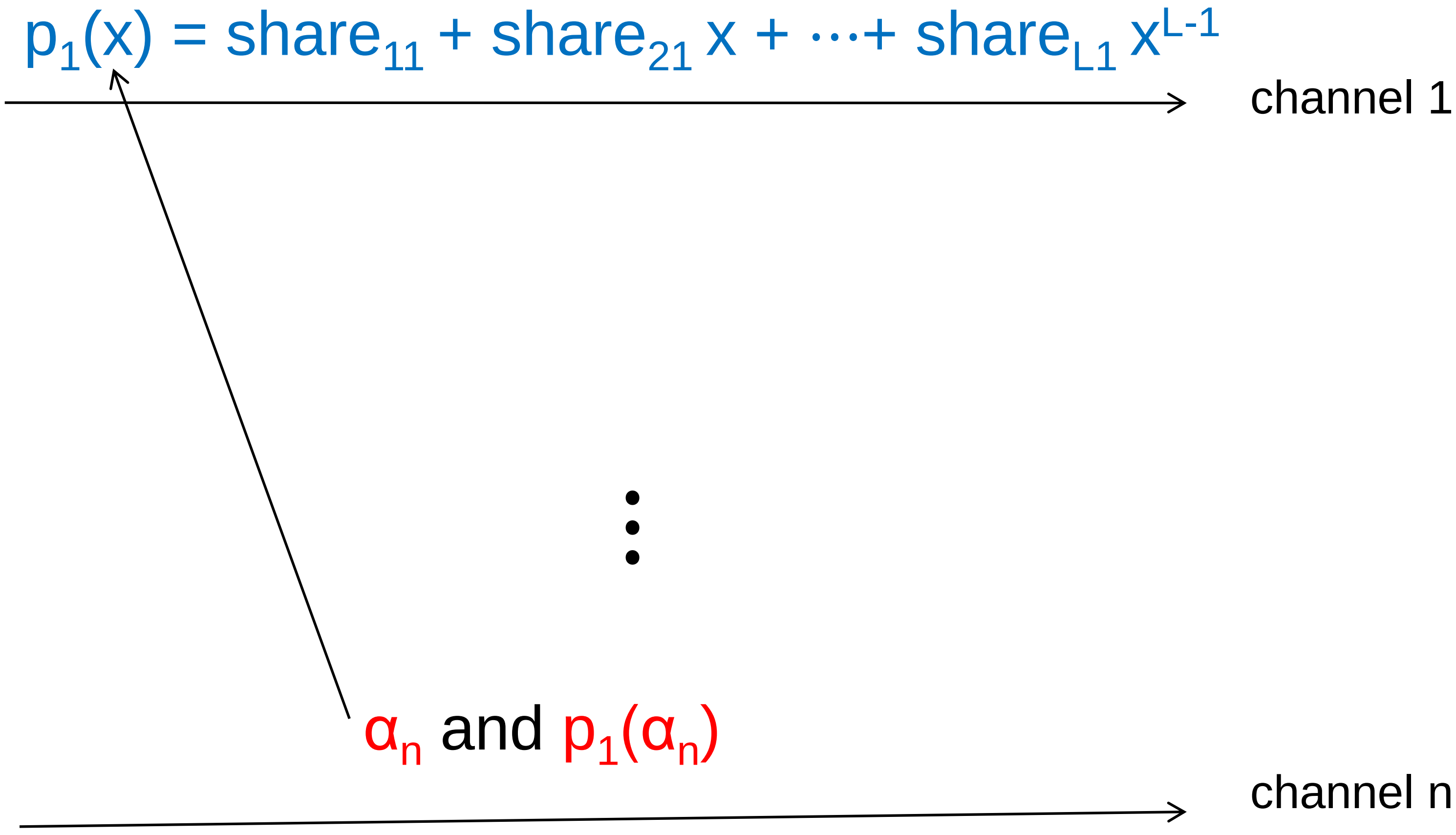
channel n

Receiver substitutes $x=\alpha_2$

$p_1(x) = share_{11} + share_{21}\, x + \cdots + share_{L1}\, x^{L-1}$



R substitutes $x=\alpha_n$



Suppose that $p_1(x)$ is forged

$$p_1(x) = share_{11} + share_{21}\, x + \cdots + share_{L1}\, x^{L-1}$$

channel 1 is corrupted

α_2 and $p_1(\alpha_2)$

channel 2 is not corrupted

$$Pr_{\alpha 2}\,[\, p_1(\alpha_2) = p_1(\alpha_2)\,] \leqq (L-1)/|F|$$

where $L-1=\deg p_1(x)$ and
the LSSS is computed over a finite field F

But

- Suppose that channel 1 is not corrupted and channel i is corrupted.
- Then

  $(\alpha_i, p_1(\alpha_i))$ leaks some information on

  $p_1(x) = share_{11} + share_{21}\, x + \cdots + share_{L1}\, x^{L-1}$

Sender hides $p_1(\alpha_i)$ as follows

$p_1(x)$ and $k_{12}, \cdots, k_{1n}$ → channel 1

α_2 and $p_1(\alpha_2)+k_{12}$ → channel 2

⋮

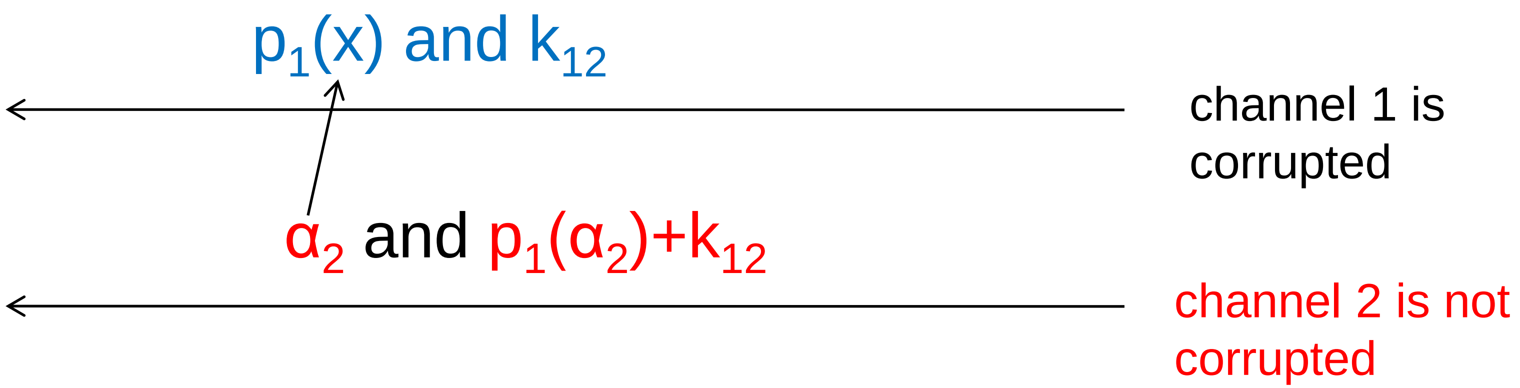
α_n and $p_1(\alpha_n)+k_{1n}$ → channel n

This is one-time pad

We do the same thing

- For $p_2(x)$, …, $p_n(x)$

Again forged $p_1(x)$ is detected



with

$$\Pr_{\alpha 2}\left[\, p_1(\alpha_2) + k_{12} \neq p_1(\alpha_2) + k_{12} \,\right] \geqq 1 - (L-1)/|F|$$

Lemma

- If $p_1(x)$ is forged,
- then

it is rejected by a correct channel i

with prob.

$$1-\frac{L-1}{|F|}$$

Next Receiver

- Reconstructs the message

  $(s_1, \ldots, s_L)$

  as follows.

Proposition

- If Γ is Q^2, then for any $B \in \Gamma$,

  $B^c \notin \Gamma$

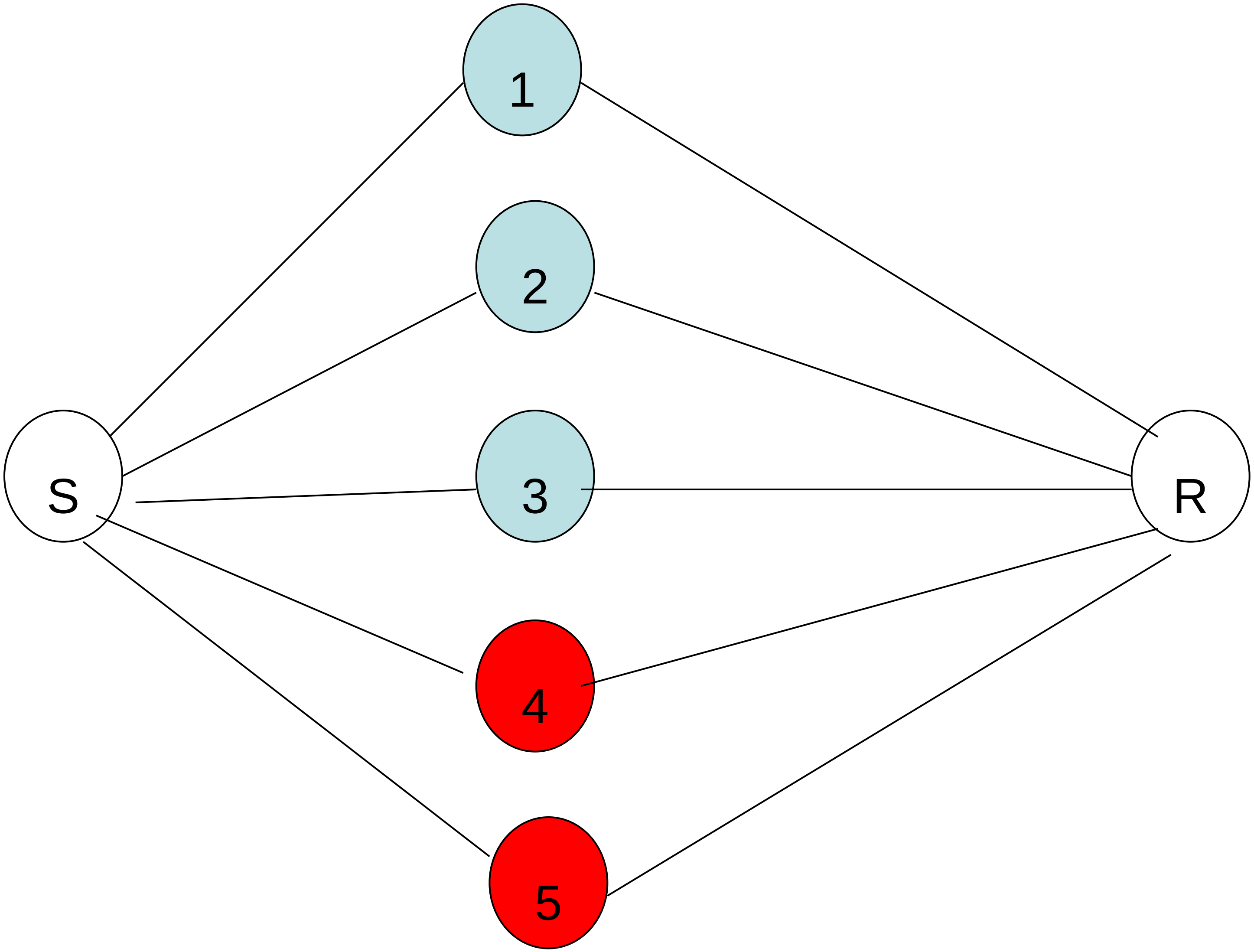
(Proof)

- Suppose that $B^c \in \Gamma$.
- Then

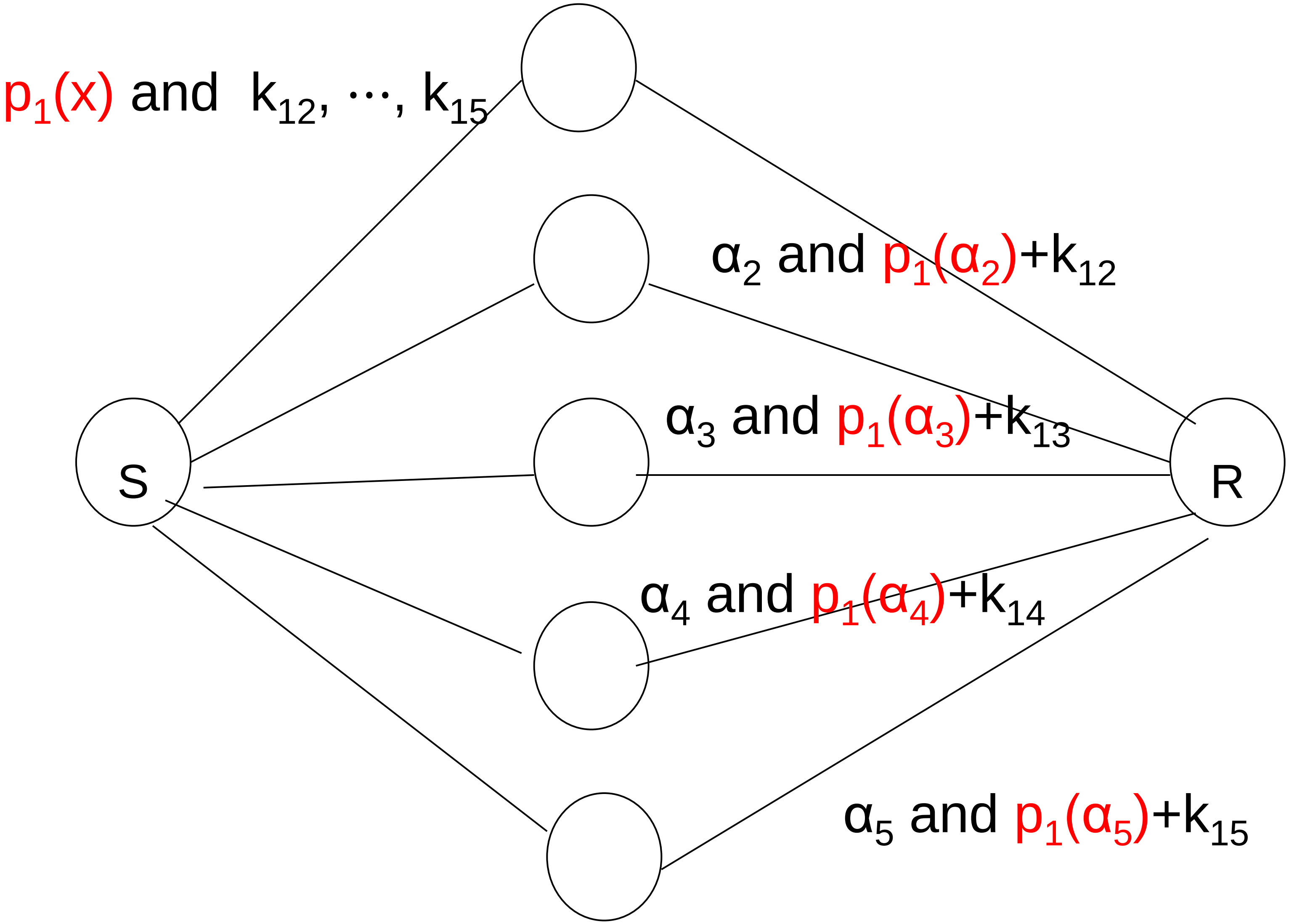
  B and $B^c \in \Gamma$

  $B \cup B^c = \{1, \cdots, n\}$
- This is against Q^2

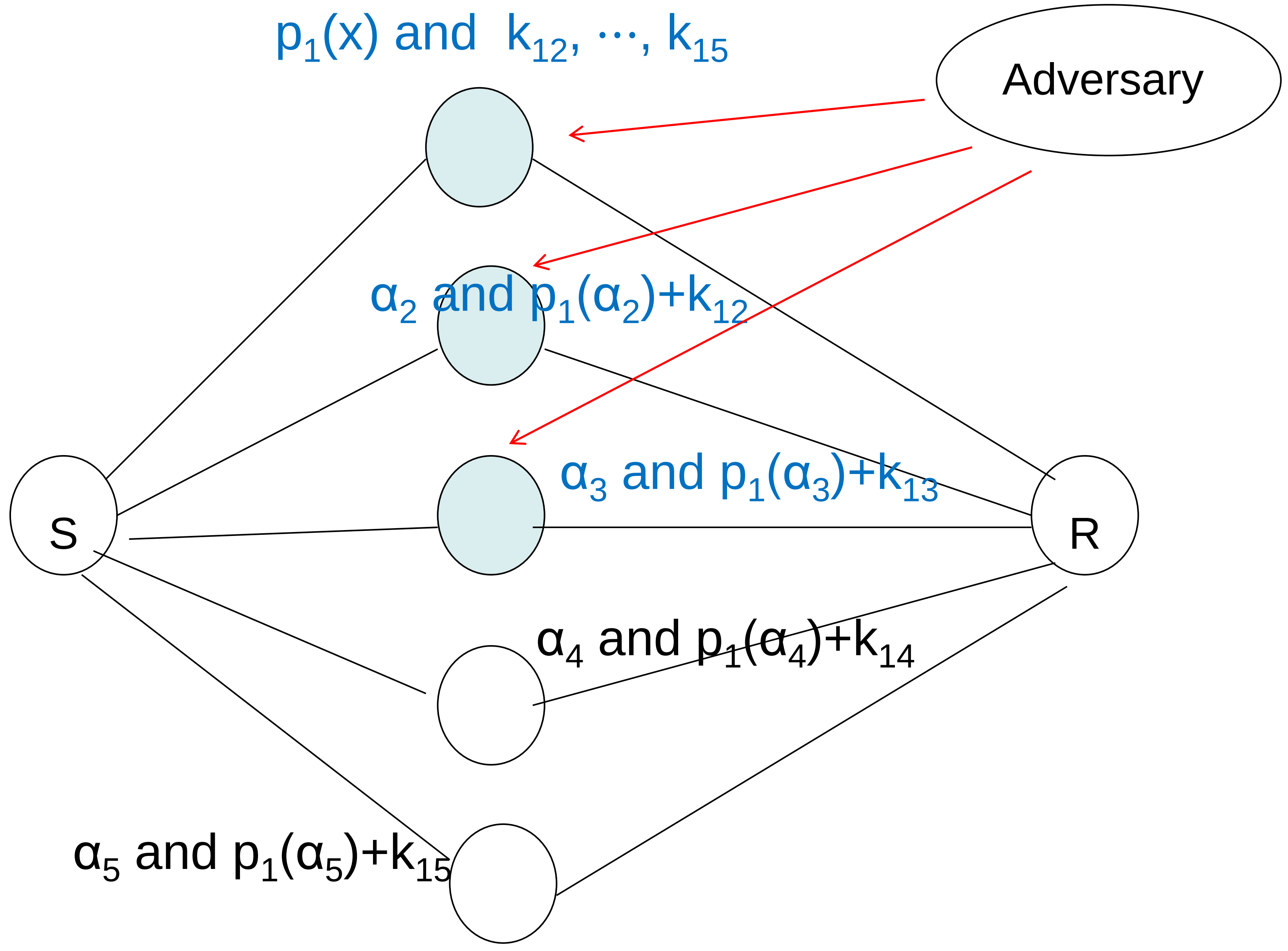
$$\{1,2,3\} \in \Gamma \text{ and } \{4,5\} \notin \Gamma$$

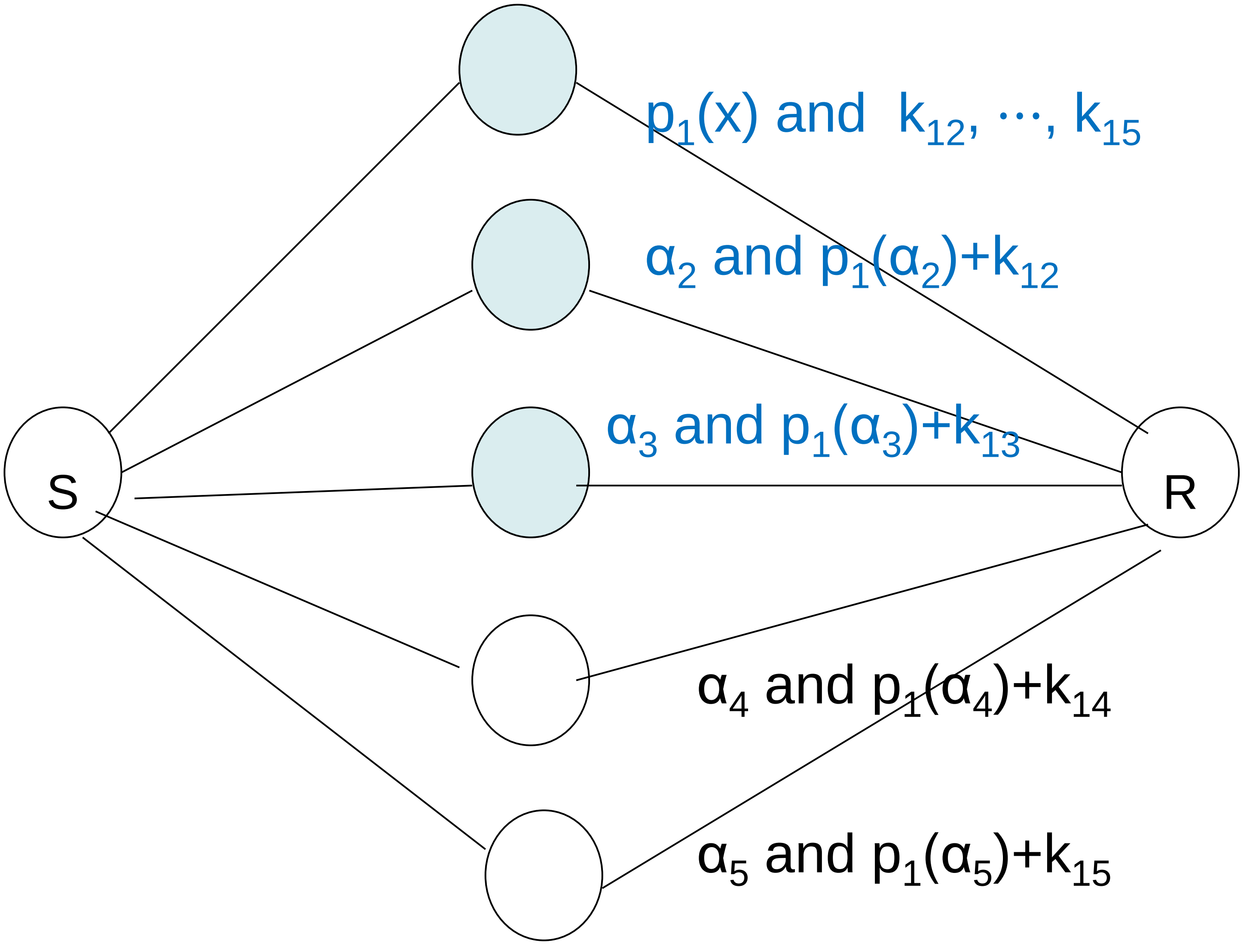


Look at $p_1(x)$

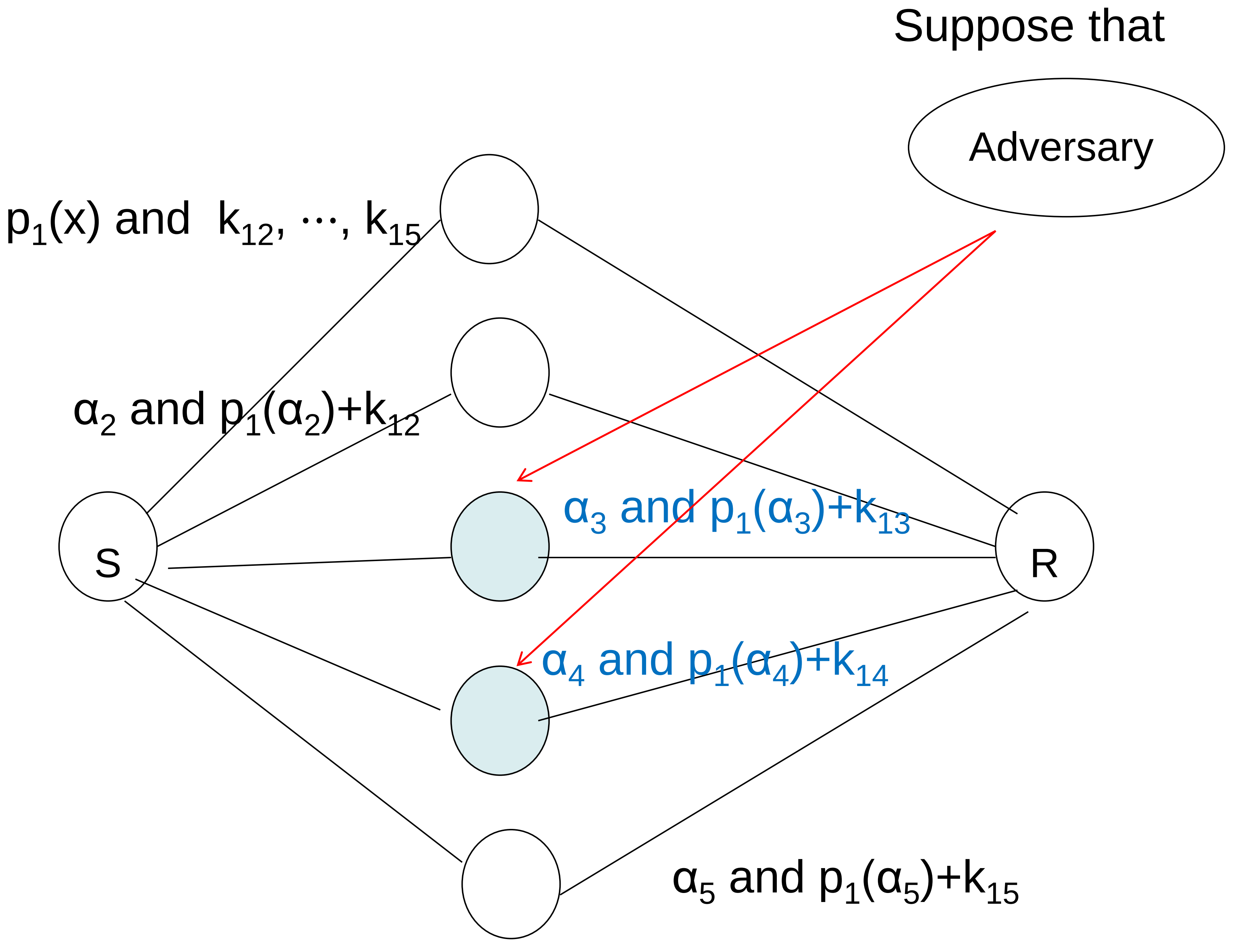

$p_1(x)$ and $k_{12}, \cdots, k_{15}$
α_2 and $p_1(\alpha_2)+k_{12}$
α_3 and $p_1(\alpha_3)+k_{13}$
S
R
α_4 and $p_1(\alpha_4)+k_{14}$
α_5 and $p_1(\alpha_5)+k_{15}$

Suppose that



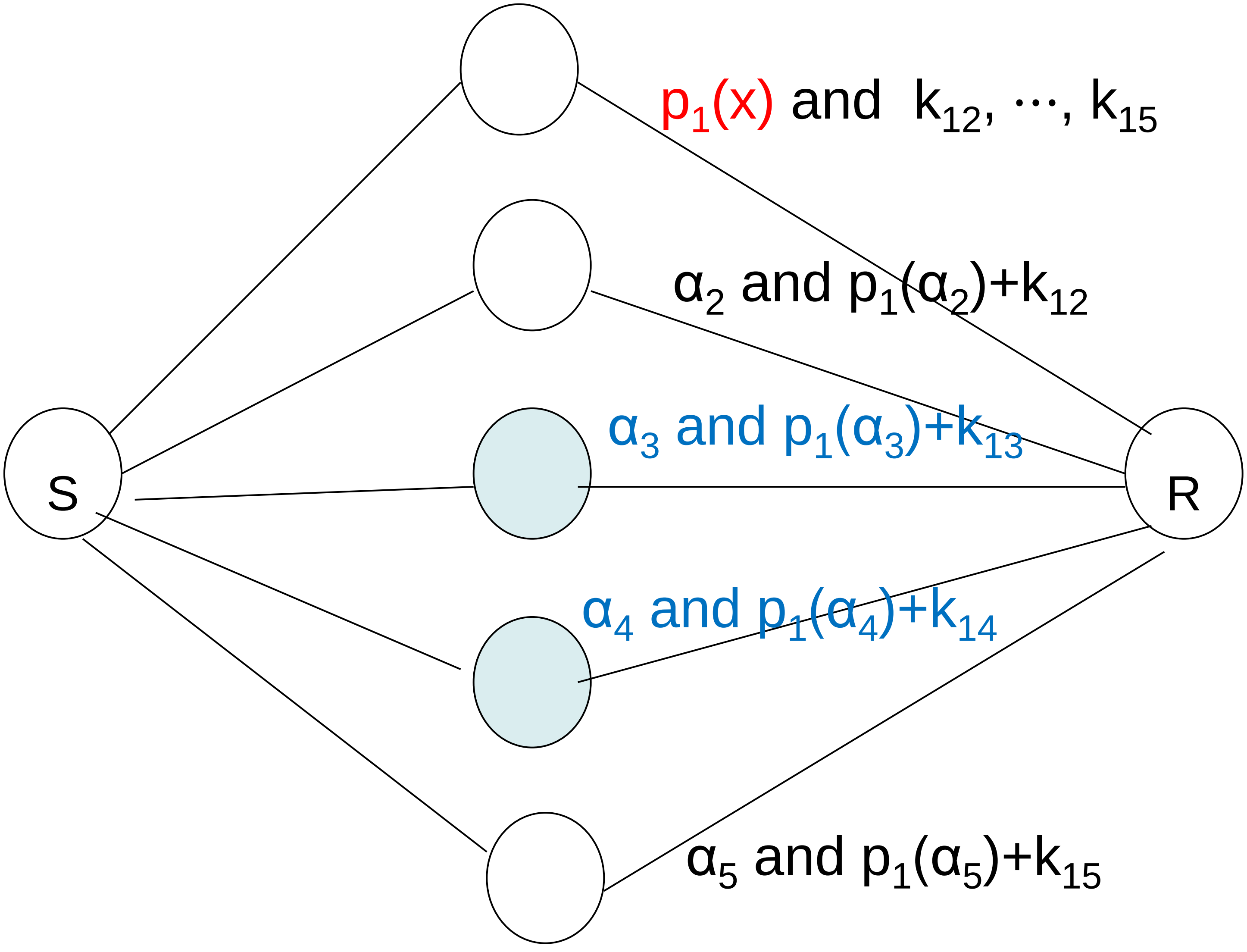


Then the forged $p_1(x)$ is rejected by channels {4 and 5} $\notin \Gamma$



Suppose that
Adversary
p1(x) and k12, ⋯, k15
α2 and p1(α2)+k12
α3 and p1(α3)+k13
S
R
α4 and p1(α4)+k14
α5 and p1(α5)+k15

In this case, $p_1(x)$ is not forged and
$p_1(x)$ is rejected by channels {3 and 4} ∈ Γ



Hence

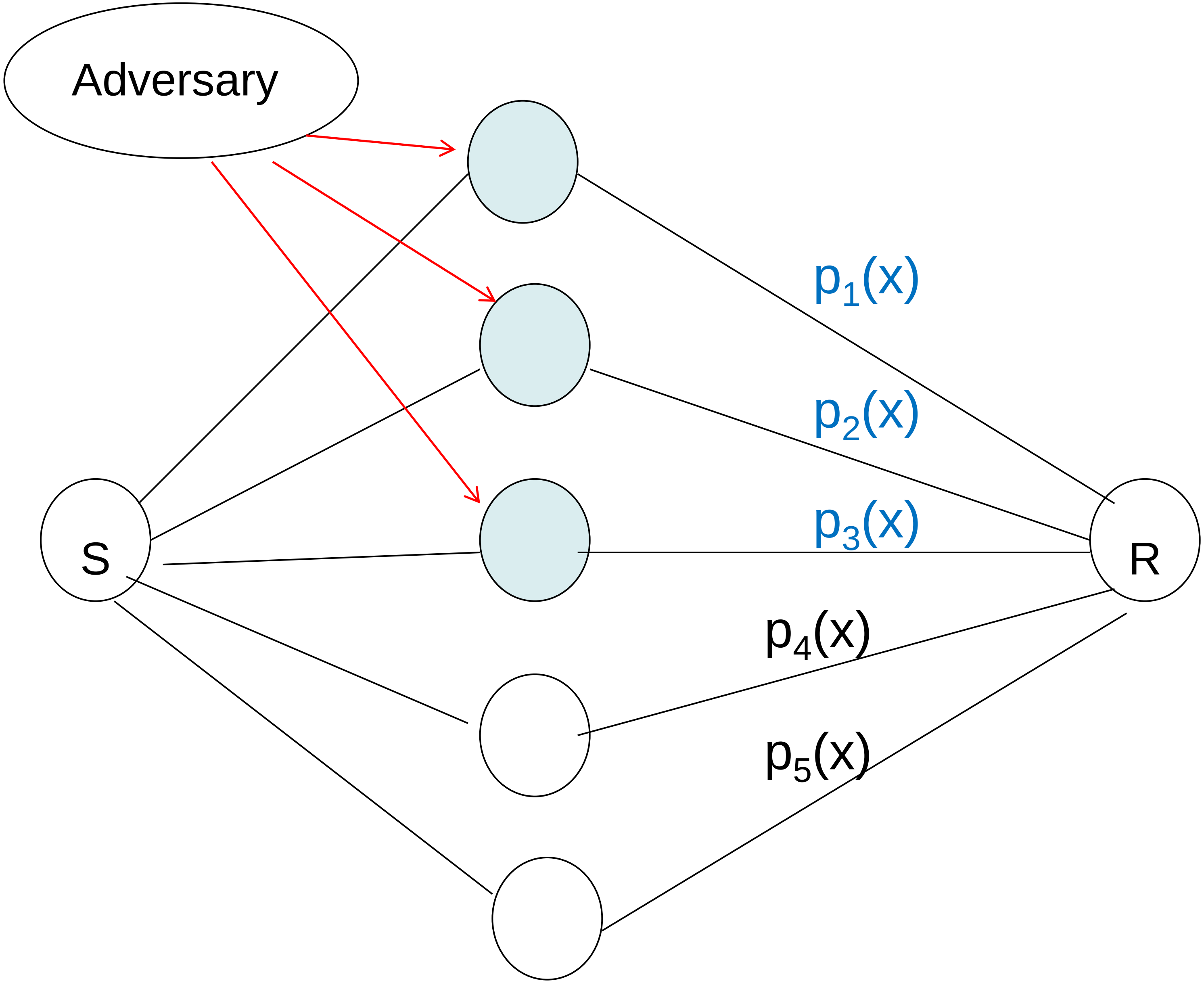
	then $p_1(x)$ is rejected
If $p_1(x)$ is forged,	by some $A \notin \Gamma$
If $p_1(x)$ is not forged,	by some $B \in \Gamma$

So Receiver behaves as follows

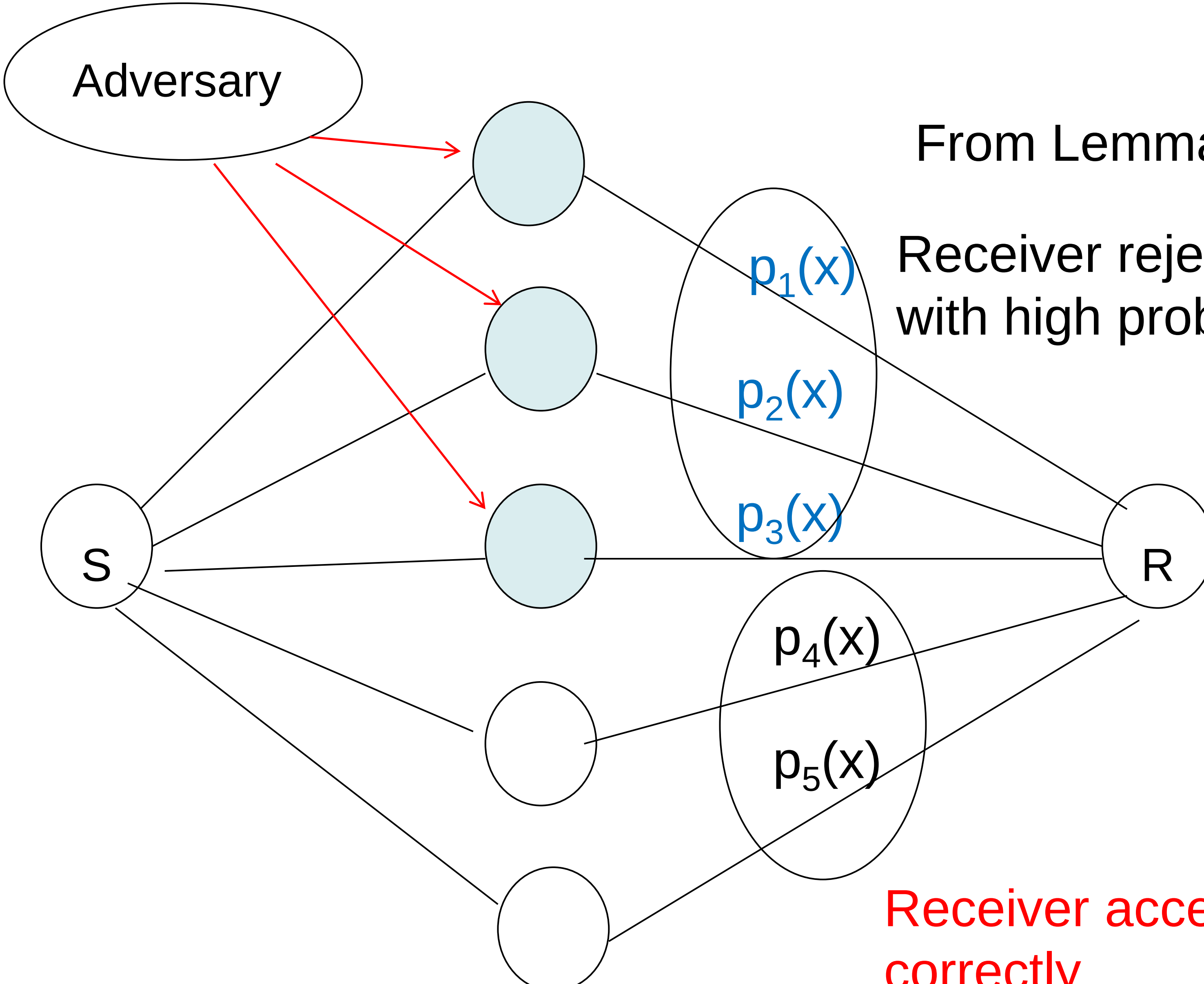
If $p_1(x)$ is rejected	Then Receiver
by some $A \notin \Gamma$	rejects $p_1(x)$
by some $B \in \Gamma$	accepts $p_1(x)$

Lemma

- If $p_1(x)$ is forged,
  R rejects it with high probability
- Otherwise
  R accepts it correctly



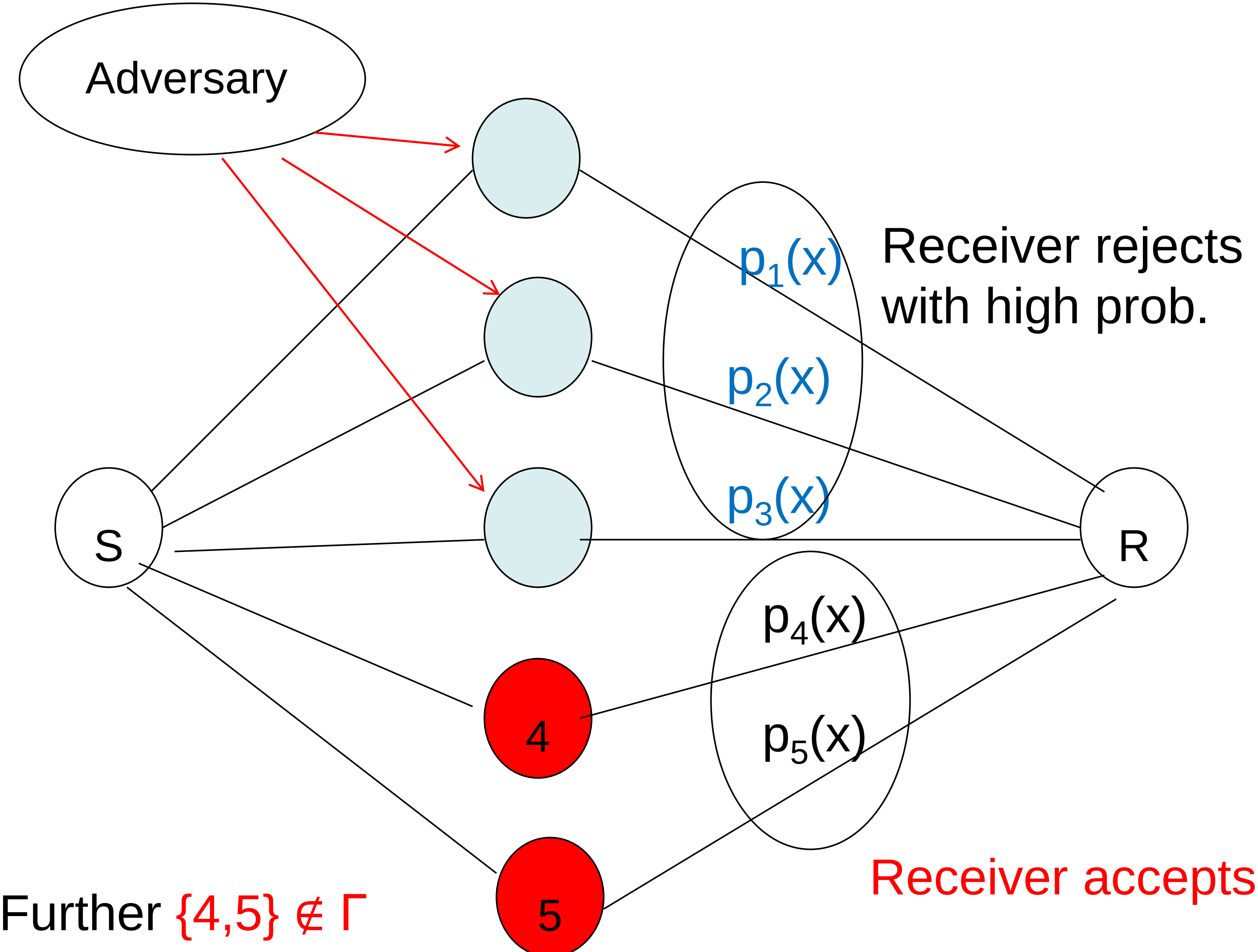
Adversary
S
R
$p_1(x)$
$p_2(x)$
$p_3(x)$
$p_4(x)$
$p_5(x)$



From Lemma,

Receiver rejects with high prob.

Receiver accepts correctly



Further {4,5} $\notin \Gamma$

Hence

{4,5} is an access set of the LSSS

Receiver accepts

$$p_4(x) = share_{14} + share_{24}\, x + \cdots + share_{L4}\, x^{L-1}$$

$$p_5(x) = share_{15} + share_{25}\, x + \cdots + share_{L5}\, x^{L-1}$$

Since {4,5} is an access set of the LSSS

$p_4(x) = share_{14} + share_{24}\, x + \cdots + share_{L4}\, x^{L-1}$

$p_5(x) = share_{15} + share_{25}\, x + \cdots + share_{L5}\, x^{L-1}$

$\downarrow$

s_1

Receiver can reconstruct

Since {4,5} is an access set

$p_4(x) = share_{14} + share_{24} x + \cdots + share_{L4} x^{L-1}$

$p_5(x) = share_{15} + share_{25} x + \cdots + share_{L5} x^{L-1}$

$\downarrow$ s_1 $\downarrow$ s_2

Receiver can reconstruct

Since {4,5} is an access set

$$p_4(x) = share_{14} + share_{24}x + \cdots + share_{L4}\, x^{L-1}$$

$$p_5(x) = share_{15} + share_{25}\, x + \cdots + share_{L5}\, x^{L-1}$$

$\downarrow$ $\qquad$ $\downarrow$ $\qquad$ $\downarrow$

s_1 $\qquad$ s_2 $\qquad$ $\cdots$ $\qquad$ s_L

Receiver can reconstruct

Theorem

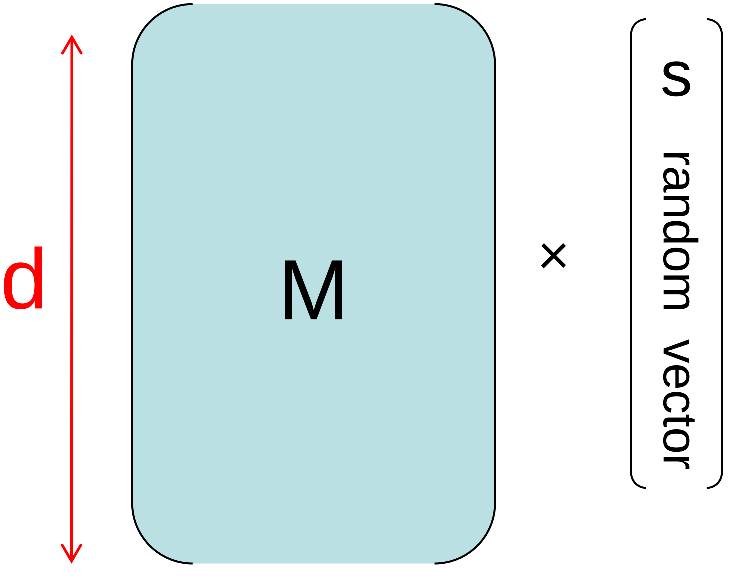
- Our protocol satisfies perfect privacy
- It also satisfies almost perfect reliability

The computational cost

- is polynomial in the size of the LSSS

The size of LSSS (=d)

is the # of rows of the matrix M



The communication cost

- Sender sends $O(Ld+d^2)$ field elements, where d is the size of the LSSS

As a special case,

- For threshold adversaries s.t. n≧2t+1, (adversary can corrupt t channels),
- our scheme is more efficient and simpler than the existing almost PSMT

Lower bound

- For threshold adversaries given by Patra, Choudhary, Srinathan and Rangan

- In any 1-round almost PSMT with $n=2t+1$, Sender must send $\Omega(nL)$ field elements to send a message $(s_1, \cdots, s_L)$

Patra et al. also showed

- A construction of
  1-round almost PSMT for n=2t+1
  which satisfies their bound

However

- It is complex
- It uses extrapolation technique, extracting randomness and etc.

Our almost PSMT

- Also satisfies the bound of Patra et al.
  if L $\geqq$ n
- Further
  it is more efficient and much simpler

Summary

We showed an efficient
1-round almost PSMT for Q^2

PSMT requires	2 rounds
Almost PSMT requires	only 1 round (This paper)

As a special case,

- For threshold adversaries s.t. $n \geqq 2t+1$,
- our scheme is more efficient and simpler than the previous almost PSMT